2n2x4/13

RESUM TÈCNIQUES BÀSIQUES de CÀLCUL de PRIMITIVES

[20-5-13]

I POTÈNCIES

$$\int x^p\,dx = \begin{cases} \dfrac{x^{p+1}}{p+1} & p \neq -1 \\ \ln|x| & p = -1 \end{cases}$$

• exemples:

▶ $\int x\,dx = \frac{1}{2}x^2 + K$ //

▶ $\int \frac{6}{x}\,dx = 6\ln|x| + K$ //

▶ $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = 2x^{1/2} + K = 2\sqrt{x} + K$ //

II CONSTANT PER FUNCIÓ

$$\int \lambda \cdot f\,dx = \lambda \int f\,dx$$

• exemples:

▶ $\int 4x\,dx = 4\int x\,dx = 4\,\frac{1}{2}x^2 + K = 2x^2 + K$ //

▶ $\int \cos 6x\,dx = \frac{6}{6}\int \cos 6x\,dx = \frac{1}{6}\int 6\cos 6x\,dx = \frac{1}{6}\cdot \operatorname{sen} 6x + K$ //

III SUMA DE FUNCIONS

$$\int (f+g)\,dx = \int f\,dx + \int g\,dx$$

• exemples:

▶ $\int \left(x + \frac{1}{x} + 2\right)dx = \int x\,dx + \int \frac{1}{x}\,dx + \int 2\,dx = \frac{1}{2}x^2 + \ln|x| + 2x + K$ //

▶ $\int \frac{2x^2 + x}{x}\,dx = \int (2x+1)\,dx = \int 2x\,dx + \int dx = x^2 + x + K$ //

IV REGLA de la CADENA

$$\int g' \cdot f'(g)\,dx = f(g)$$

• exemples:

▶ $\int 2x\,e^{x^2}\,dx = e^{x^2} + K$ //

▶ $\int \cos x \cdot \operatorname{sen}^3 x\,dx = \frac{1}{4}\operatorname{sen}^4 x + K$ //

▶ $\int \operatorname{tg} x\,dx = \int \operatorname{sen} x \,\frac{1}{\cos x}\,dx = -\ln|\cos x| + K$ //

V CAS CONCRET R. CADENA

$$\int f' \cdot f\,dx = \frac{1}{2}f^2$$

• exemples:

▶ $\int \operatorname{sen} x \cdot \cos x\,dx = \frac{1}{2}\operatorname{sen}^2 x + K$ //

▶ $\int \frac{\operatorname{arctg} x}{1+x^2}\,dx = \frac{1}{2}\operatorname{arctg}^2 x + K$ //

VI INTEGRAL PER PARTS

$$\int uv'\,dx = uv - \int vu'\,dx$$

• exemples:

▶ $\int x e^x\,dx = \left\| \begin{matrix} u = x \to u' = 1 \\ v' = e^x \to v = e^x \end{matrix} \right\| = xe^x - \int e^x\,dx = e^x(x-1) + K$ //

▶ "recursiva":

$$\underbrace{\int \cos^2 x\,dx}_{I} = \left\| \begin{matrix} u = \cos x \to u' = -\operatorname{sen} x \\ v' = \cos x \to v = \operatorname{sen} x \end{matrix} \right\| = \cos x \cdot \operatorname{sen} x + \int \operatorname{sen}^2 x\,dx =$$

($\operatorname{sen}^2 x + \cos^2 x = 1$)

$$= \cos x \cdot \operatorname{sen} x + \underbrace{\int dx}_{x+K} - \underbrace{\int \cos^2 x\,dx}_{I} \Rightarrow$$

Ajuntem I:

$$I = \int \cos^2 x\,dx = \frac{1}{2}(\cos x \operatorname{sen} x + x) + C$$ //