ESCOLA PIA Sabadell

M2 - tecno

« PROVA integrals indef: solucions »

[dc; 5-III-14]

(1) Immediates:

1.- $\int(-9x^5 + x^3 + \frac{2}{7})\,dx = -\frac{3}{2}x^6 + \frac{1}{4}x^4 + \frac{2}{7}x + K$

2.- $\int(7-2x)^2\,dx = \frac{1}{-2\cdot 3}\int(-2\cdot 3)\cdot(7-2x)^2\,dx =$

$= -\frac{1}{6}(7-2x)^3 + K$

3.- $\int 5x\,e^{1+x^2}\,dx = \frac{5}{2}\int 2x\,e^{1+x^2}\,dx = \frac{5}{2}e^{1+x^2} + K$

4.- $\int(x^2+1)\sin(2x^3+6x)\,dx = -\frac{1}{6}\int(-1)\cdot(6x^2+6)\sin(2x^3+6x)\,dx =$

$= -\frac{1}{6}\cos(2x^3+6x) + K$

5.- $\int(1-x)\frac{1}{\sqrt[3]{x}}\,dx = \int\left(x^{-1/3} - x^{1-1/3}\right)dx =$

$= \int\left(x^{-1/3} - x^{2/3}\right)dx = \frac{3}{2}x^{2/3} - \frac{3}{5}x^{5/3} + K$

6.- $\int\left[2\cos(2x+2) - \frac{2}{3x}\right]dx = \int 2\cos(2x+2)\,dx - \frac{2}{3}\int\frac{1}{x}\,dx =$

$= \sin(2x+2) - \frac{2}{3}\ln|x| + K$

7.- $\int\frac{\cos 7x}{\sin^2 7x}\,dx = \frac{1}{7}\int 7\cos 7x\,(\sin 7x)^{-2}\,dx =$

$= -\frac{1}{7}(\sin 7x)^{-1} + K = \frac{-1}{7\sin 7x} + K$

8.- $\int\frac{-x}{2\sqrt{x}}\,dx = -\frac{1}{2}\int x^{1-\frac{1}{2}}\,dx = -\frac{1}{2}\int x^{1/2}\,dx =$

$= -\frac{1}{2}\frac{2}{3}x^{3/2} + K = -\frac{1}{3}x^{3/2} + K$

$$\int u\,dv = uv - \int v\,du \quad (*)$$

② Per parts:

9.- $\int \ln x \, dx = \left\| \begin{matrix} u = \ln x & \rightarrow & du = \frac{1}{x}\,dx \\ dv = dx & \rightarrow & v = x \end{matrix} \right\| =$

$= x \ln x - \int \frac{x}{x}\,dx = x\ln x - \int dx = (*,*)$

([*]; també: $\int =$ )

$[*,*] = x \ln x - x + K = x(\ln x - 1) + K$ //

10.- $\int x^2 \cos x \, dx = \left\| \begin{matrix} u = x^2 & \rightarrow & du = 2x\,dx \\ dv = \cos x\,dx & \rightarrow & v = \sin x \end{matrix} \right\| \overset{[*]}{=}$

$= x^2 \sin x - 2\underbrace{\int x \sin x \, dx}_{} = \left\| \begin{matrix} u = x & \rightarrow & du = dx \\ dv = \sin x\,dx & \rightarrow & v = -\cos x \end{matrix} \right\| =$

[*] $= -x\cdot\cos x + \int \cos x\,dx$

$= x^2 \sin x - 2\cdot\left\{ -x\cdot\cos x + \underbrace{\int \cos x\,dx}_{\sin x\,+\,\text{constant}} \right\} =$

$= x^2 \sin x + 2x\cos x - 2\sin x + K$ //