《57 derivades》

▶ DERIVADES pendents del tema $\boxed{1.1}$: (7 derivades)

· 40) $y = -\frac{7}{2}x^2 + \frac{2}{7}x^3 \rightarrow y' = -7x + \frac{6}{7}x^2$

· 41) $y = ax + b \rightarrow y' = a$

· 42) $y = \sin x \rightarrow y' = \cos x$

· 43) $y = \underbrace{\sin(-x)}_{= -\sin x} \rightarrow y' = \boxed{-\cos(-x)} = -\cos x$

· 44) $y = -\sin x \rightarrow y' = -\cos x$

· 45) $y = \underbrace{-\sin(-x)}_{= \sin x} \rightarrow y' = \boxed{\cos(-x)} = \cos x$

· 46) $y = -\cos x \rightarrow y' = \sin x$

▷ Derivades pendents del tema $\boxed{1.2}$: (20 derivades)

· 68) $y = (-9x^2 + x + 3)^5 \rightarrow y' = 5(-9x^2 + x + 3)^4(-18x + 1)$

· 69) $y = x^5 \cos^6 x \rightarrow y' = 5x^4 \cos^6 x - 6x^5 \cos^5 x \sin x$

· 70) $y = 6x^3 \sin^2 x \rightarrow y' = 18x^2 \sin^2 x + 12x^3 \sin x \cos x$

· 71) $y = \frac{1}{x^5} - \cos^2 x \rightarrow y' = -\frac{5}{x^6} + 2 \cos x \sin x$

· 72) $y = \frac{1}{\sqrt{x}} + 2\cos^5 x = x^{-\frac{1}{2}} + 2\cos^5 x \rightarrow$

$\rightarrow y' = -\frac{1}{2\sqrt{x^3}} - 10 \cos^4 x \sin x$ ▨

· 73) $y = x^{7/8} - \sqrt{x} \cos x \rightarrow$

$\rightarrow y' = \frac{7}{8} x^{-1/8} - \frac{1}{2\sqrt{x}} \cos x + \sqrt{x} \sin x =$

$= \frac{7}{8\sqrt[8]{x}} - \frac{\cos x}{2\sqrt{x}} + \sqrt{x} \sin x$ ▨

· 74) $y = \frac{2 \sin 5x}{\sqrt{x}} \rightarrow y' = 2\, \frac{5 \cos 5x \cdot \sqrt{x} - \sin 5x \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x})^{\cancel{2}}} =$

$= \frac{10\, x \cos 5x - \sin 5x}{x\sqrt{x}}$ ▨

$\frac{\sqrt{x}}{\sqrt{x}} \cdot$

· 75) $y = \sqrt[5]{x} \cos(1-x) \rightarrow y' = \frac{1}{5} x^{-4/5} \cos(1-x) +$

$+ \sqrt[5]{x} \cdot (-\sin(1-x)) \cdot (-1) = \frac{1}{5\sqrt[5]{x^4}} \cos(1-x) + \sqrt[5]{x} \sin(1-x)$ ▨

· 76) $y = \sqrt{5x} \rightarrow y' = \dfrac{5}{2\sqrt{5x}}$

· 77) $y = \sqrt{4x+1} \rightarrow y' = \dfrac{4}{2\sqrt{4x+1}} = \dfrac{2}{\sqrt{4x+1}}$ ▨

· 78) $y = \dfrac{1}{2x^5 - 1} \rightarrow y' = -\dfrac{10x^4}{(2x^5-1)^2}$

· 79) $y = \dfrac{4}{\sqrt{x^3 - x}} = 4(x^3-x)^{-1/2} \rightarrow$

$\rightarrow y' = -4\,\dfrac{1}{2}\cdot(x^3-x)^{-\frac{1}{2}-\frac{2}{2}}\cdot(3x^2-1) = \dfrac{2-6x^2}{\sqrt{(x^3-x)^3}}$ ▨

· 85) $y = \dfrac{x}{\cos x} \rightarrow y' = \dfrac{\cos x + x\sin x}{\cos^2 x}$

· 86) $y = \dfrac{\sqrt{x}}{\sin x} \rightarrow y' = \dfrac{\frac{1}{2\sqrt{x}}\sin x - \sqrt{x}\cos x}{\sin^2 x} =$

$\dfrac{2\sqrt{x}}{2\sqrt{x}}\cdot$

$= \dfrac{\sin x - 2x\cos x}{2\sqrt{x}\,\sin^2 x}$ ▨

· 87) $y = \dfrac{14x^3 - 2x^2 + 22x}{2x} = \dfrac{14x^3}{2x} - \dfrac{2x^2}{2x} + \dfrac{22x}{2x} =$

$= 7x^2 - x + 11 \rightarrow y' = 14x - 1$ ▨

· 89) $y = \dfrac{x^3 - 3x^2 + 7}{x+3} \rightarrow$

$$\rightarrow y' = \frac{(3x^2 - 6x)\cdot(x+3) - (x^3 - 3x^2 + 7)}{(x+3)^2} =$$

$$= \frac{3x^3 + 9x^2 - 6x^2 \boxed{-18x} - x^3 + 3x^2 - 7}{(x+3)^2} =$$

$$= \frac{2x^3 + 6x^2 - 18x - 7}{(x+3)^2}$$

· 91) $y = \dfrac{\cos x}{1+\sqrt{x}} \rightarrow y' = \dfrac{-\sin x\cdot(1+\sqrt{x}) - \cos x \cdot \frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2} =$ $\frac{2\sqrt{x}}{2\sqrt{x}}\cdot$

$$= -\frac{2\sqrt{x}\,\sin x\cdot(1+\sqrt{x}) + \cos x}{2\sqrt{x}\,(1+\sqrt{x})^2}$$

· 92) $y = \dfrac{Ax^2 + B}{m - x} \rightarrow y' = \dfrac{2Ax(m-x) + (Ax^2+B)}{(m-x)^2} =$

$$= \frac{2Amx - 2Ax^2 + Ax^2 + B}{(m-x)^2} = \frac{-Ax^2 + 2Amx + B}{(m-x)^2}$$

· 101) $y = \dfrac{1}{\sqrt{x}} + x^2 \operatorname{tg} x = x^{-1/2} + x^2 \operatorname{tg} x \rightarrow$

$$\rightarrow y' = -\frac{1}{2}x^{-3/2} + 2x\operatorname{tg}x + x^2\frac{1}{\cos^2 x} = \frac{-1}{2\sqrt{x^3}} + 2x\operatorname{tg}x + \frac{x^2}{\cos^2 x}$$

· 102) $y = \sqrt{5x}\,\sin^2(x^2) \rightarrow$

$$\rightarrow y' = \frac{5}{2\sqrt{5x}}\sin^2(x^2) + 4x\sqrt{5x}\,\sin(x^2)\cdot\cos(x^2)$$

▶ DERIVADES del tema $\boxed{1.3}$: (26 derivades)

a) $f'(x) = 1$ b) $f'(x) = 3x^2$ c) $f'(x) = 6\cdot 3x^2 + 2\cdot 2x - 1 =$
$= 18x^2 + 4x - 1$ d) $f'(x) = \dfrac{1\cdot(x+1) - x\cdot(1)}{(x+1)^2} = \dfrac{x+1-x}{(x+1)^2} = \dfrac{1}{(x+1)^2}$

e) $f'(x) = \dfrac{(10x+2)\cdot(3x^4 - x^3) - (12x^3 - 3x^2)\cdot(5x^2+2x)}{(3x^4 - x^3)^2} =$

$$= \frac{30x^5 - 10x^4 + 6x^4 - 2x^3 - (60x^5 + 24x^4 - 15x^4 - 6x^3)}{(3x^4 - x^3)^2} =$$

$$= \frac{30x^5 - 4x^4 \boxed{-2x^3} - 60x^5 - 24x^4 + 15x^4 \boxed{+6x^3}}{(3x^4 - x^3)^2} = \frac{-30x^5 - 13x^4 + 4x^3}{(3x^4 - x^3)^2} = (*)$$

[traiem factor comú x^3 a dalt, i també x^3 a dins del parèntesi de baix, que surt com a x^6 (pel quadrat)]

$$[*] = \frac{-30x^2 - 13x + 4}{(3x-1)^2\ x^6}\, x^3 = \frac{-30x^2 - 13x + 4}{(3x-1)^2\ x^3}$$

f) $f'(x) = 3\cdot(6x+5)^2\cdot 6 = 18(6x+5)^2$ g) $f'(x) = (x^{-5})' = -5x^{-6} = -\dfrac{5}{x^6}$

h) $f'(x) = 6\cdot(-\operatorname{sen} x) = -6\operatorname{sen} x$ i) $f'(x) = 3\cdot\operatorname{sen}^2 x\cdot\cos x$

j) $f'(x) = 9\cdot tg^8(x)\cdot(1 + tg^2 x) = 9\cdot tg^8 x + 9\cdot tg^{10} x$; equivalentment:
$f'(x) = 9\cdot tg^8(x)\cdot\dfrac{1}{\cos^2 x} = 9\cdot tg^8 x\cdot \sec^2 x$

k) $f'(x) = 10\cdot 2\cdot\operatorname{sen} x\cdot\cos x = 20\operatorname{sen} x\cos x$

l) $f'(x) = 14\cdot(\cos^{-2} x)' =$

$= 14\cdot(-2)\cdot\cos^{-3} x\cdot(-\operatorname{sen} x) = 28\,\dfrac{\operatorname{sen} x}{\cos^3 x}$; equivalentment: $f'(x) = 28\, tg x \sec^2 x$

[donc $\dfrac{\operatorname{sen} x}{\cos x} = tg x$; $\dfrac{1}{\cos x} = \sec x$]

m) $f'(x) = \cancel{2}\frac{1}{\cancel{2}\sqrt{x}} = \frac{1}{\sqrt{x}}$ // n) $f'(x) = \frac{1}{2\sqrt{4x+5}} \cdot 4 = \frac{2}{\sqrt{4x+5}}$ //

o) $f'(x) = 5 \cdot \frac{1}{2\sqrt{\cos x}} \cdot (-\operatorname{sen} x) = -\frac{5 \operatorname{sen} x}{2\sqrt{\cos x}}$ // p) $f'(x) = 7\left(\left(\sqrt{x}\right)^{-1}\right)' =$

$= 7 \cdot (-1) \cdot (\sqrt{x})^{-2} \cdot \frac{1}{2\sqrt{x}} = -7 \frac{1}{(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}} = -\frac{7}{2} \cdot \frac{1}{x\sqrt{x}} = -\frac{7}{2} \cdot \frac{1}{x \cdot x^{1/2}} = -\frac{7}{2x^{3/2}}$ //

q) $f'(x) = \frac{5}{x}$ // r) $f'(x) = 7 \cdot \ln^6 x \cdot \frac{1}{x} = \frac{7 \ln^6 x}{x}$ //

s) $f'(x) = \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{\ln x}}$ // t) $f'(x) = 2e^x$ //

u) $f'(x) = \frac{1}{2\sqrt{e^x}} \cdot e^x = \frac{\sqrt{e^x}}{2}$ // v) $f'(x) = e^x + \frac{1}{x}$ //

w) $f'(x) = \frac{\frac{1}{x} \cdot (1+x^2) - 2x \cdot \ln x}{(1+x^2)^2} = \frac{1 + x^2 - 2x^2 \ln x}{x\,(1+x^2)^2}$ //

[multipliquem a dalt i a baix per x, per treure el $\frac{1}{x}$ del numerador]

x) $f'(x) = \cos x \cdot \ln x + \operatorname{sen} x \cdot \frac{1}{x}$ //

y) $f'(x) = (1 + \operatorname{tg}^2 x) \cdot \sqrt{x} + \operatorname{tg} x \, \frac{1}{2\sqrt{x}}$;

equivalentment: $f'(x) = \frac{1}{\cos^2 x} \sqrt{x} + \operatorname{tg} x \, \frac{1}{2\sqrt{x}} = \frac{2x \sec^2 x + \operatorname{tg} x}{2\sqrt{x}}$ //

$\sec^2 x \cdot \sqrt{x} = \frac{2x \sec^2 x}{2\sqrt{x}}$

z) $f'(x) = a \cdot 2x + b + 0 = 2ax + b$ //